Element exchange search algorithm for optimal placement of viscous dampers for building structures

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ABSTRACT

Optimal design of supplemental dampers efficiently enhances the seismic performance of structures. Existing methods of optimal damper placement design either require a large amount of iteration or can only reliably arrive at locally optimum solution. This paper describes the transfer of Element Exchange Method (EEM) from topology optimisation research to the optimisation of viscous damper placement. Moreover, a new search algorithm, Inverse Element Exchange Method (IEEM) is proposed in this paper. These two innovative methods provide engineers with converging quickly to the globally optimal damper placement. This paper presents the design procedure of the methods. The effectiveness of these novel methods is validated by comparing that with the effectiveness of commonly-used methods including uniform distribution, SSSA, Genetic Algorithm and other distribution methods.

1 INTRODUCTION

The addition of viscous dampers can reduce the engineering demand of structures due to earthquakes. As computing advances in the last twenty years, enabled complicated calculations for engineering design are conducted more easily. Many studies proposed different optimal placement design of viscous dampers. There are three categories of viscous damper placement design methods. These are evolutionary methods, analytical approaches and heuristic approach(De Domenico, Ricciardi, & Takewaki, 2019).

Evolutionary methods, Genetic Algorithm (GA) used in this study, is according to biology evolution patterns. Its advantage is to set different beginning points and carry out the sequential search without calculating any gradient and be able to address optimisation problems of non-differentiable objective functions varying steeply within all design variables. In the process of Genetic Algorithm, there are five key steps containing: (1) Setting the initial population (2) determining the fitness function of all the population (3) Selection (4) Crossover (5) Mutation. (Hejazi, Toloue, Jaafar, & Noorzaei, 2013; Movaffaghi & Friberg, 2006; Singh & Moreschi, 2001; Singh, Moreschi, & dynamics, 2002) employed Genetic Algorithm to allocate viscous dampers along the buildings' height.

Analytical approaches, as the name suggests, demand a series of analyses and iterations to search the optimal solution. The advantage of them is able to take shorter time and appropriate iterations but still with rigorous optimisation procedure to search by using science-based modelling and analyses. In terms of analytical approaches, (Takewaki, 1997, 2000) proposed a way for obtaining optimal damper placement subjected to the constraint of the sum of damping coefficient to minimise the sum of the amplitude of the transfer function using the gradient-based method. (Levy & Lavan, 2006) proposed another optimization methodology for placing dampers was the fully-stressed analysis/ redesigned procedure, i.e., Lavan A/R method utilising the recurrence relationship to distribute the damping coefficient of viscous dampers.

Heuristic approaches characterise structural seismic behaviour in simplified way or enable the damper design process to be easier. Heuristic approaches contain the commonly-used method, Uniform Distribution (UD), Story Shear Proportional Distribution (SSPD) (Pekcan, Mander, & Chen, 1999), Distribution Based on Story Shear Strain Energy (SSSE), Distribution Based on Story Shear Strain Energy to Efficient Stories (SSSEES) (Hwang, Huang, Yi, & Ho, 2008), Distribution based on Energy Dissipated by Viscous Dampers (EDVD), Distribution Based on Energy Dissipated by Viscous Dampers to Efficient Stories (EDVDES) (Chan, 2016) apportioning the damping coefficient proportionally according to one importance factor (e.g. story shear, story shear strain energy and energy dissipated by viscous dampers, etc.) represented by the fundamental mode shape of buildings. A simplified optimisation way for allocating dampers, e.g., Simplified Sequential Search Algorithm (SSSA) belonging to heuristic approaches, was proposed (Garcia, 2001; Lopez Garcia & Soong, 2002) performing time history analysis in each iteration and utilising peak inter-storey drifts or velocity as performance indices to allocate one damper to the storey with maximum the performance index along the building height in every iteration until all dampers are allocated to the building.

The aforementioned methods have some drawbacks. For example, even if Genetic Algorithm is able to arrive at the global optimal solution, it demands a large amount of time and appropriate initial populations of chromosomes to acquire the optimal solution. SSSA can use a simple concept and procedure to design viscous damper positioning but trials and iterations increase as the number of dampers increases and SSSA is sensitive to the ground motion used for design. Therefore, this study is to propose innovative methods which take an appropriate time and iteration obtaining ideal seismic performance aiming to offer alternative approaches for viscous damper placement design for the engineering practice.

2 ELEMENT EXCHANGE SEARCH ALGORITHMS

2.1 Element Exchange Method (EEM)

(Rouhi, Rais-Rohani, & Williams, 2010) proposed Element Exchange Method (EEM) which is able to be utilised to optimise the design of damper placement. In each iteration of structural topology optimisation, EEM removes the least utilised element and adds the removed element to the highest utilised location. In the same way, this research applies EEM concept to remove the viscous dampers in the storey with less objective functions to the storey with higher ones to minimise the objective functions.

This study utilises the peak inter-storey drift ratio as the objective function to minimise and uses total damping coefficient as the constraint. Thus, the optimisation procedure of EEM is setting up the initial positioning of viscous dampers and move dampers from stories with minimum drift ratios to stories with maximum drift ratios. The EEM process is as follows:

1. Set up the initial positioning of viscous dampers which adopts UD herein.
2. Carry out dynamic analysis to obtain the objective function (i.e. the peak inter-storey drift ratio).
3. Check the inter-storey drift ratio and move one damper from the storey with minimum drift ratio to that with maximum one.

4. Repeat from step 2 to step 3 until the objective function of the next iteration in one direction is not smaller than that of the current iteration.

The optimisation problem can be expressed as follows:

$$\begin{gathered} \min \max\{(\delta_1)_{max}, \cdots (\delta_n)_{max}\} \text{ , variables : } c_1, \cdots c_n \\ \text{s.t. } \sum_{i=1}^{n} c_i = C \\ c_i \geq 0, \qquad i = 1, \cdots, n \\ c_i \in \{0, c, 2c, \cdots pc\} \end{gathered} \tag{1}$$

where $(\delta_i)_{max}$ is the maximum inter-storey drift ratio of *i* th story, *n* is total stories, c_i is the damping coefficient at *i* th story, *C* is the sum of damping coefficients which is the constraint, p is an integer greater than zero.

2.2 Inverse Element Exchange Method (IEEM)

This study proposes an innovative method known as Inverse Element Exchange Method (IEEM). IEEM is able to overcome the drawbacks of EEM since EEM do not consider the effectiveness of the combination of damper placement at the next iterative step. In other words, IEEM takes into account the intervention of viscous damper allocated for each next step of iteration and is capable of searching the steepest gradient of the objective function at each step of the iteration which results in the optimal positioning of viscous dampers. IEEM herein utilises the same objective function and constraint as EEM uses. The EEM process is as follows:

1. Set up the initial positioning of viscous dampers which adopts UD herein.
2. Carry out dynamic analysis to obtain the objective function (i.e. the peak inter-storey drift ratio).
3. Check the inter-storey drift ratio and move one damper from the storey with minimum drift ratio to all the other storey. If the building has n storey number, the number of candidate damper configurations is n-1.
4. Carry out dynamic analysis for all candidate damper configuration and acquire the objective function (i.e. the peak inter-storey drift ratio) corresponding to all candidate.
5. Select the candidate damper configuration with the minimum objective function among all candidates as the damper design.
6. Repeat from step 2 to step 5 until the objective function of the next iteration in one direction is not smaller than that of the current iteration.

The optimisation problem of IEEM can be expressed in eq. (1).

3 CASE STUDY BUILDING AND GROUND MOTION SELECTION

3.1 Case study structure

This study designs a 8-storey RC MRF building according to New Zealand standards (Standard, 1995, 2004) and based on the red book (Bull & Brunsdon, 1998). The plan and elevation of the building is shown in Figure 1 and Figure 2. The structure is assumed to be symmetric meaning that the centre of rigidity (CR) and the centre of mass (CM) is located at the same point over all stories. L1, L2, L3, and L4 represent the bay allowed to place viscous dampers. The height of each storey is 3.3 m.

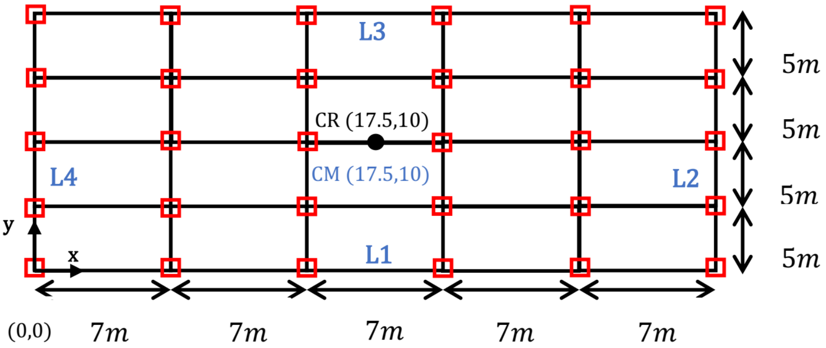

Figure 1: The plan of the case study structure

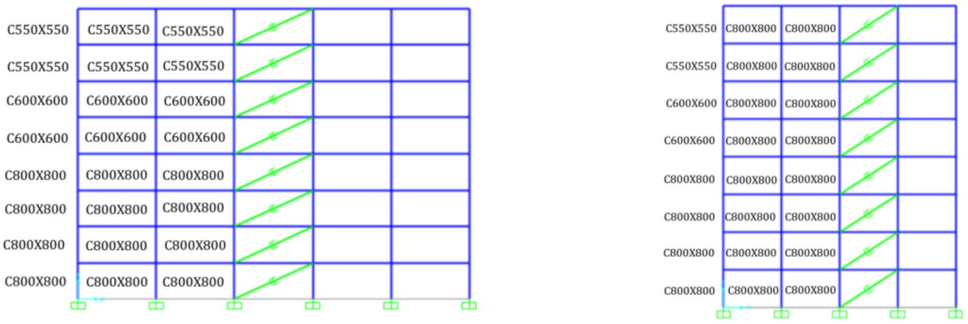

Figure 2: The elevation of the case study structure

The column section is shown is Figure 2. Since the structure is symmetric, Figure 2 only shows the half part of the dimension of column section. It is noteworthy that Figure 2 only shows the column section of the exterior frame. The column section of interior frames is 500 mm by 500 mm over the building height. The material of concrete and rebar as well as the dimension of beam and slab are shown in Table 1 and Table 2 respectively.

Table 1: The structure material

Material	**Strength**
Rebar	$f_y = 500\ Mpa$ (Gr500E)
Rebar (for shear and confinement)	$f_y = 500\ Mpa$ (Gr300E)
Concrete	$f'_c = 40\ Mpa$

Table 2: Dimension of beam and slab

Section	**Size**
External Beam	$600 \times 400\ mm$
Internal Beam	$400 \times 300\ mm$
Slab thickness	150mm

3.2 Ground motion selection

Ten pairs of ground motion records were selected from the Pacific Earthquake Engineering Research (PEER) Centre's NGA database. The ground motions were selected according to magnitude, distance from the fault to site and site condition. The magnitude of these ground motions varies from 6.5 to 7.9 and these selected ground motions were recorded at 22-198 km from the closest point of the fault rupture and on the site class C of NZS1170. The selected ground motions were scaled to the target spectrum at serviceability limit state over the period range of interest based on NZS1170.5. The pairs of ground motion are applied to the analysis as acceleration in the principal axes, these are subsequently swapped to ensure both combinations of ground motion directions are tested. This resulted in 20 time history analyses for each test since each pair of ground motion records has two orthogonal components which are exchanged for application along x and y directions, respectively.

4 COMPARISON OF PROPOSED METHODS WITH EXISTING METHODS

This study adopts Simplified Sequential Search Algorithm (SSSA), Genetic Algorithm, and distribution methods including Distribution based on Story Shear Strain Energy (SSSE), Distribution based on Story Shear Strain Energy to Efficient Stories (SSSEES) proposed by (Hwang et al., 2008), Distribution based on Energy Dissipated by Viscous Dampers (EDVD), Distribution based on Energy Dissipated by Viscous Dampers to Efficient Stories (EDVDES) proposed by (Chan, 2016) and Uniform Distribution. To be fair to compare the effectiveness of all methods, the total damping coefficient is used as the constraint for all methods to design damper placements.

4.1 The existing methods used in the case study

4.1.1 Simplified Sequential Search Algorithm (SSSA)

The procedure of the SSSA could be conceptually described as follows: At first, the peak inter-story drift ratio of the bare structure (without any supplemental dampers), which is obtained by time-history analyses, is used as optimal location index and the greatest optimal location indicates the optimal placement of the first damper. Then, the second damper is placed at the story where the new optimal location index, which takes the first added damper into account. After that, the procedure is repeated until all dampers have been placed one by one in the structure.

4.1.2 Genetic Algorithm (GA)

This study adopts 50 initial populations, Roulette Wheel Selection (RWS) as the selection strategy, 70% as the probability for crossover, and 20% as the probability for mutation for GA. The objective function is combined with the penalty function as the fitness function shown below:

$$\Phi = f_{obj} + p \tag{2}$$

$$f_{obj} = \max(\delta_x)^2 + \max(\delta_y)^2 \tag{3}$$

$$p = \left(\sum_j C_j - \sum_i C_i\right)^2 \tag{4}$$

Where Φ is the objective function, f_{obj} is the fitness function, p is the penalty function, δ_x and δ_y are the maximum peak inter-storey drift ratio in x and y direction, $\sum_j C_j$ is the constraint of the total damping coefficient and $\sum_i C_i$ is the total damping coefficient of one specific population.

The fitness function of each chromosome is used as criteria to calculate the probability to select the chromosome with the corresponding fitness function into the crossover pool based on RWS in each iteration.

4.1.3 Uniform Distribution (UD)

UD distributes damping coefficients uniformly at each story. Based on the equivalent damping ratio, the damping coefficient contributed by linear viscous dampers at each story can be expressed as

$$c_i = \frac{4\pi\xi_d \sum_i m_i \varphi_i^2}{T \sum_j \varphi_{rj}^2 cos^2\theta_j} \tag{5}$$

where θ_i is the inclination angle of the damper on *i* th storey to the horizontal, φ_{rj} is the first relative mode shape, c_i is the damping coefficient of the *i* th storey, *T* is the fundamental structural period.

In addition, the sum of the damping coefficients can be described as follows:

$$\sum_i c_i = C \tag{6}$$

4.1.4 Distribution based on Story Shear Strain Energy (SSSE)

The concept of this distribution is the sum of the damping coefficients is distributed according to the story shear strain energy relationship E_i at each story. The story shear strain energy relationship at each story can express as

$$E_i \propto \varphi_{ri} \sum_{j=i}^{n} m_j \varphi_j \tag{7}$$

where E_i is the storey shear strain energy of *i* th storey, *n* is the total number of storeys of the building, m_j is the mass on *j* th storey and φ_{ri} is the value of the first relative mode shape on *i* th storey.

Then, the damping coefficient distribution formula can be expressed as

$$c_i = \frac{E_i}{\sum_j E_j} C \tag{8}$$

4.1.5 Distribution based on Story Shear Strain Energy to Efficient Stories (SSSEES)

In order to make more efficient use of viscous dampers, the total damping coefficient is distributed only to those stories with a shear strain energy larger than the average story shear strain energy.

$$\varphi_{ri} S_i > \frac{\sum_{j=1}^{n} \varphi_{rj} S_j}{n} \tag{9}$$

Then, the damping coefficient distribution based on the SSSE to efficient story (SSSEES) can be derived as

$$c_i = \frac{E_i}{\sum_j E_j} C \tag{10}$$

where *i* and *j* are the stories with a shear strain energy larger than the average story shear strain energy.

4.1.6 Distribution based on Energy Dissipated by Viscous Dampers (EDVD)

When a MDOF system with viscous dampers is subjected to a sinusoidal excitation, the work done by those dampers in a cycle can be expressed as follows

$$W_D = \sum_j W_{Dj} = \sum_j \lambda_j c_j \omega^\alpha (u_j)^{1+\alpha} = \sum_j \lambda_j c_j \omega^\alpha A^{1+\alpha} cos^{1+\alpha}\theta_j \varphi_{rj}^{1+\alpha} \tag{11}$$

Hence, the distribution formula can be expressed as

$$c_i = \frac{W_{Di}}{\sum_j W_{Dj}} C = \frac{\lambda_i c_i \omega^\alpha A^{1+\alpha} cos^{1+\alpha}\theta_i {\varphi_{ri}}^{1+\alpha}}{\lambda_j c_j \omega^\alpha A^{1+\alpha} cos^{1+\alpha}\theta_j \sum_j {\varphi_{rj}}^{1+\alpha}} C = \frac{{\varphi_{ri}}^{1+\alpha}}{\sum_j {\varphi_{rj}}^{1+\alpha}} C \tag{12}$$

where $\lambda = 2^{2+\alpha} \frac{\Gamma^2(1+\frac{\alpha}{2})}{\Gamma(2+\alpha)}$, Γ is gamma function, θ_i, θ_j is the inclination angle of the damper on *i* th and *j* th storey to the horizontal, respectively, φ_{ri} is the value of first relative mode shape on *i* th storey, c_i is the damping coefficient of the i th storey, ω is first mode natural frequency, A is the maximum roof displacement, α is damping exponent which is equal to 1 in this study.

4.1.7 Distribution based on Energy Dissipated by Viscous Dampers to Efficient Stories (EDVDES)

The total damping coefficient is distributed only to those stories with a relative mode shape to the power of α larger than the average relative mode shape to the power of α.

$$ {\varphi_{ri}}^{1+\alpha} > \frac{\sum_{j=1}^{n} {\varphi_{rj}}^{1+\alpha}}{n} \tag{13} $$

Then, the damping coefficient distribution based on the EDVD to the efficient story (EDVDES) can be derived as

$$ c_i = \frac{{\varphi_{ri}}^{1+\alpha}}{\sum_j {\varphi_{rj}}^{1+\alpha}} C \tag{14} $$

where i and j are the stories with a story drift larger than the average story drift.

4.2 Viscous damper placement design of all methods

The inherent damping ratio of the building is 5% and the equivalent damping ratio supplemented by viscous dampers is 15% in both horizontal directions herein. Thus, the total damping coefficient in the x-direction C_x is 189010.88 kN-s/m and in the y-direction C_y is 247159.36 kN-s/m. Since the number of dampers is assumed to be two times of the number of storeys, the damping coefficient of each damper in the x-direction c_x is 11813.18 kN-s/m and in the y-direction c_y is 15447.46 kN-s/m. One limitation of EEM, IEEM, SSSA and GA is the ground motion records selected to conduct the optimisation process. Thus, a single ground motion record selected for EEM, IEEM, SSSA, and GA to design damper placement is an issue due to the sensitivity of each method of ground motion. Five pairs of ground motion records are used herein to design and to observe variations in the placement due to different ground motions. Figure 3, Figure 4, Figure 5, and Figure 6 demonstrate damper allocation designs of the aforementioned methods due to five pairs of ground motions. The variation of each method in damper placements due to different ground motion is minimal. NGA2116 selected herein to design damper placements which can result in the most frequently occurring damper placement design is chosen for the design ground motion.

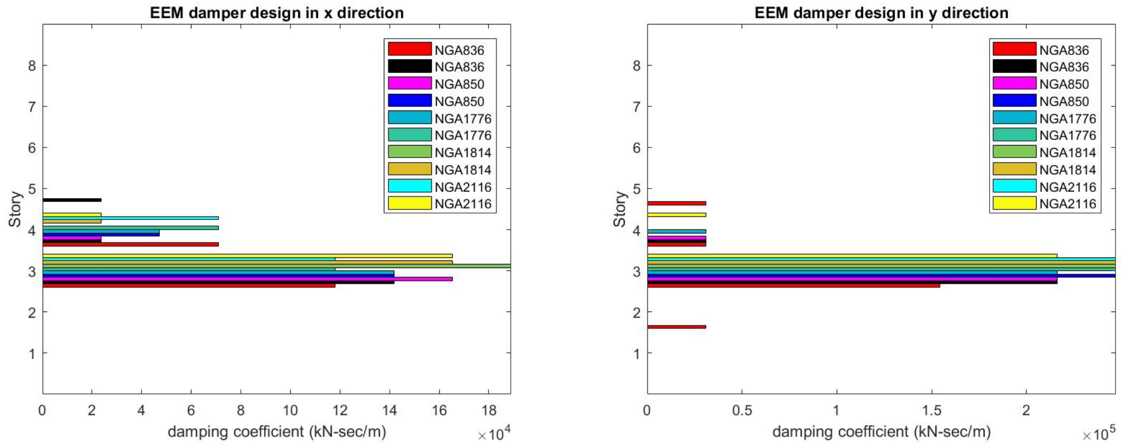

Figure 3: The damper placement design of EEM due to five pairs of ground motions

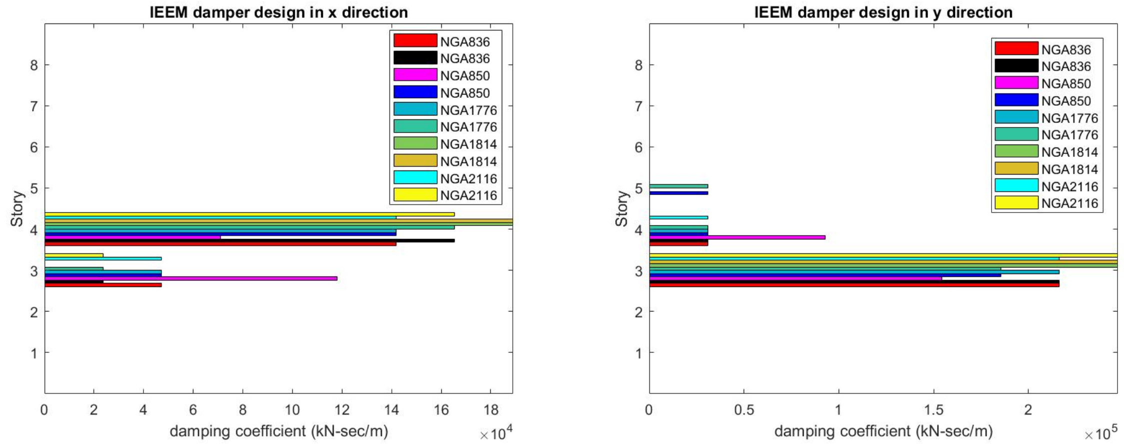

Figure 4: The damper placement design of IEEM due to five pairs of ground motions

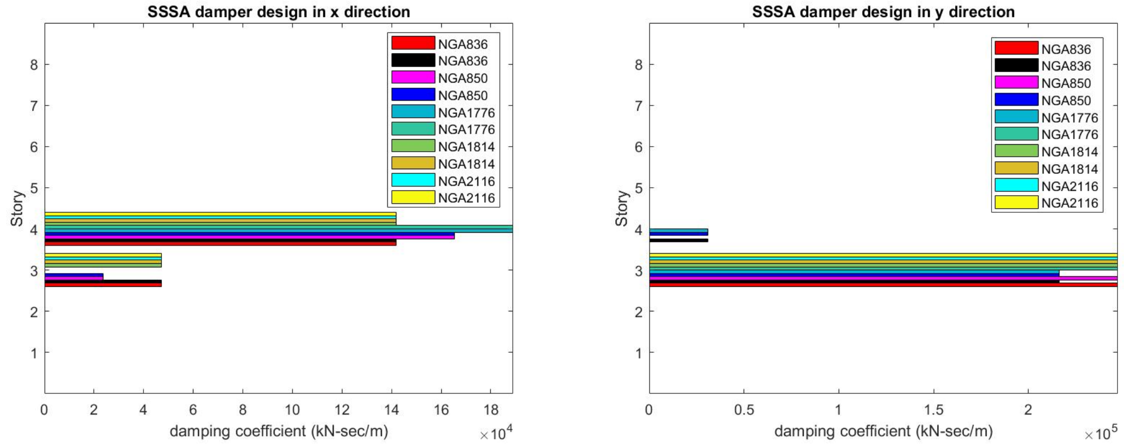

Figure 5: The damper placement design of SSSA due to five pairs of ground motions

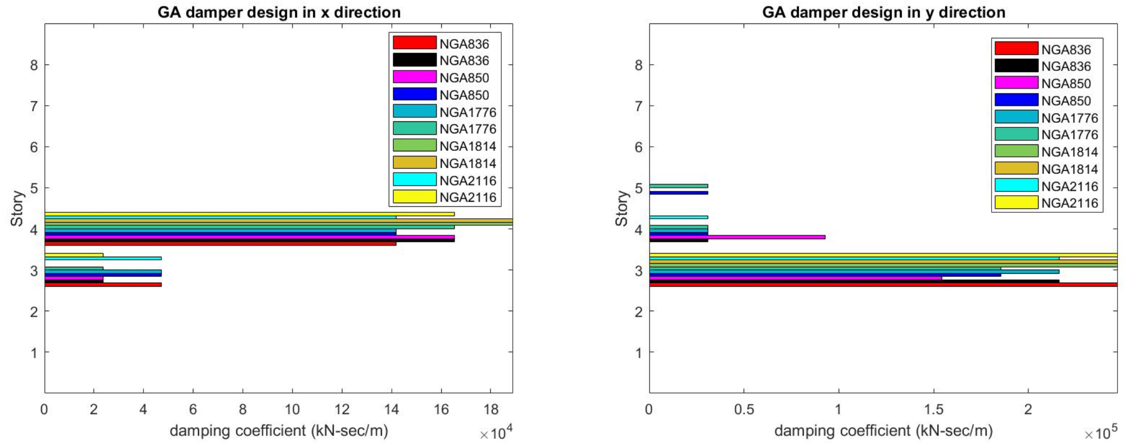

Figure 6: The damper placement design of GA due to five pairs of ground motions

The damping coefficients supplemented by viscous dampers of each storey of each method are shown in Figure 7. As can be seen in Figure 7, The methods requiring carrying out time history analyses with some iterations (i.e. EEM, IEEM, SSSA, GA) tend to concentrate the dampers on the third and fourth storey where the maximum inter-storey drift ratio often occurs. Moreover, IEEM and GA acquire the same damper design in both horizontal directions.

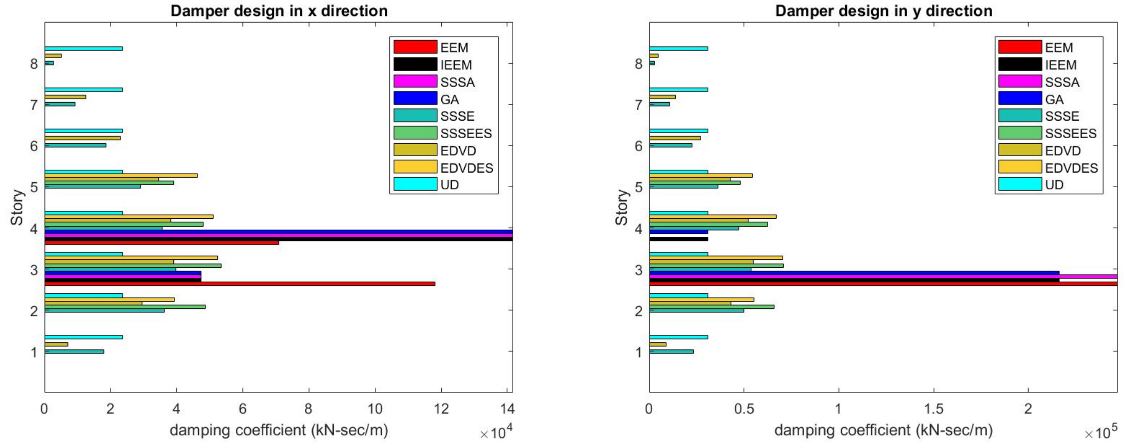

Figure 7: The damper placement design of each storey of all methods

4.3 Engineering demand parameter results

Figure 8 and Figure 9 reveals the engineering demand parameters (EDPs) of the building utilising EEM and IEEM damper design. It shows that EEM can reduce peak floor acceleration in x-direction better than IEEM and the differences in terms of other EDPs is minimal. Figure 10 and Figure 11 take into account the EDPs of the building with the viscous damper placement designed by existing methods. As can be observed in Figure 10 and Figure 11, methods requiring time history analyses and iteration perform well in terms of the peak inter-storey drift ratio (IDR) in both horizontal directions. SSSEES and EDVDES can mitigate the peak floor acceleration and obtain similar acceleration results as the methods requiring time history analyses do. It is noteworthy that even though IEEM, SSSA, GA can mitigate the peak IDR in the x-direction, they are not capable of controlling peak floor acceleration on the top of the building. Table 3 and Table 4 shows the mean and standard deviation of maximum EDPs under each ground motion in the x-direction. Table 3 and Table 4 indicate that except EEM, IEEM, SSSA, GA performing better than the rest of methods, EDVDES can also enable to make the building to reduce the mean of peak EDPs without any time history analysis and iterations.

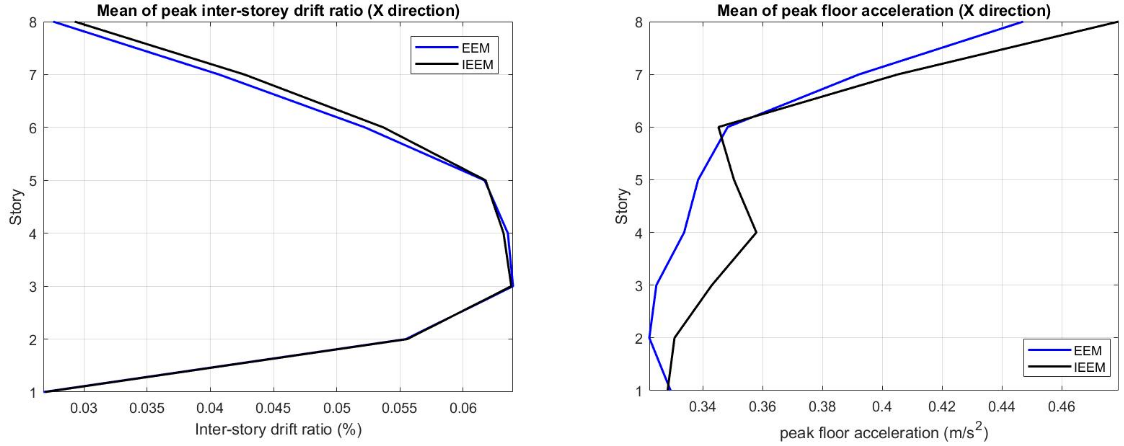

Figure 8: Mean of peak inter-storey drift ratio and floor acceleration in the x-direction of EEM and IEEM

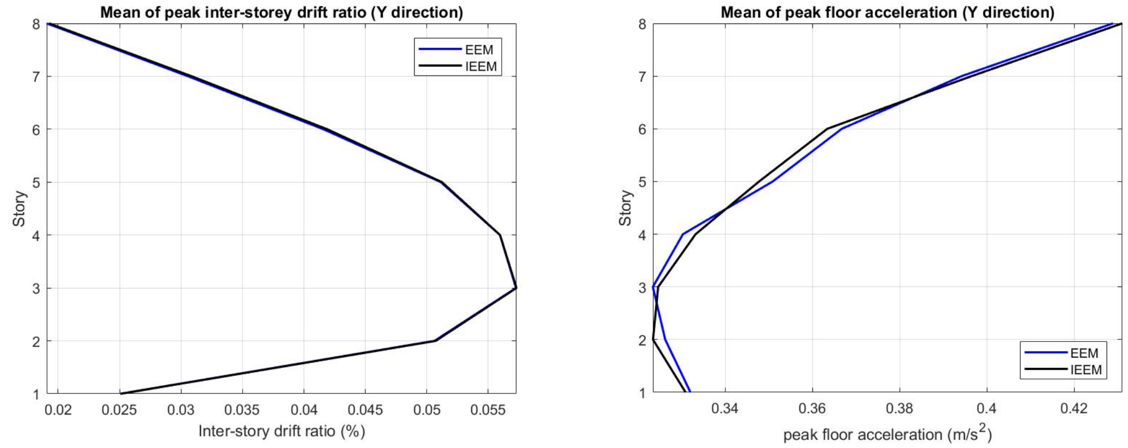

Figure 9: Mean of peak inter-storey drift ratio and floor acceleration in the y-direction of EEM and IEEM

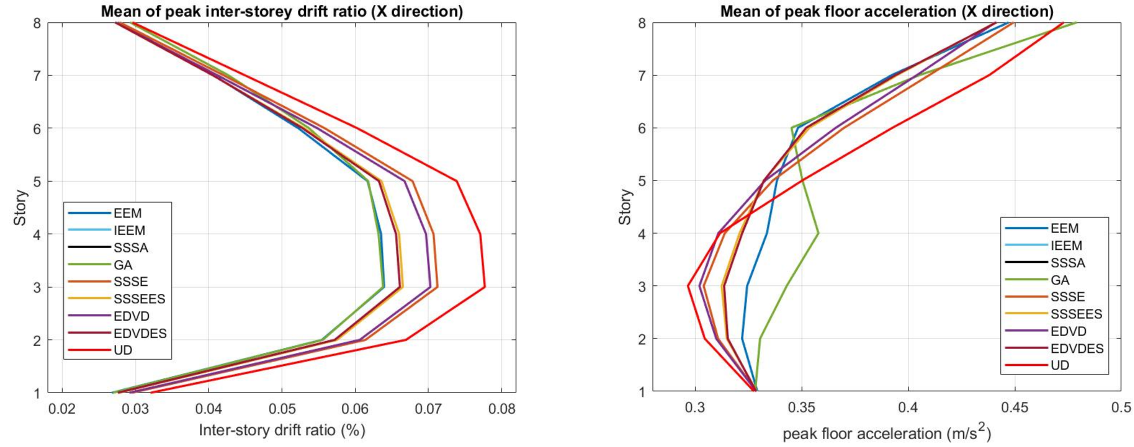

Figure 10: Mean of peak inter-storey drift ratio and floor acceleration in the x-direction of all methods

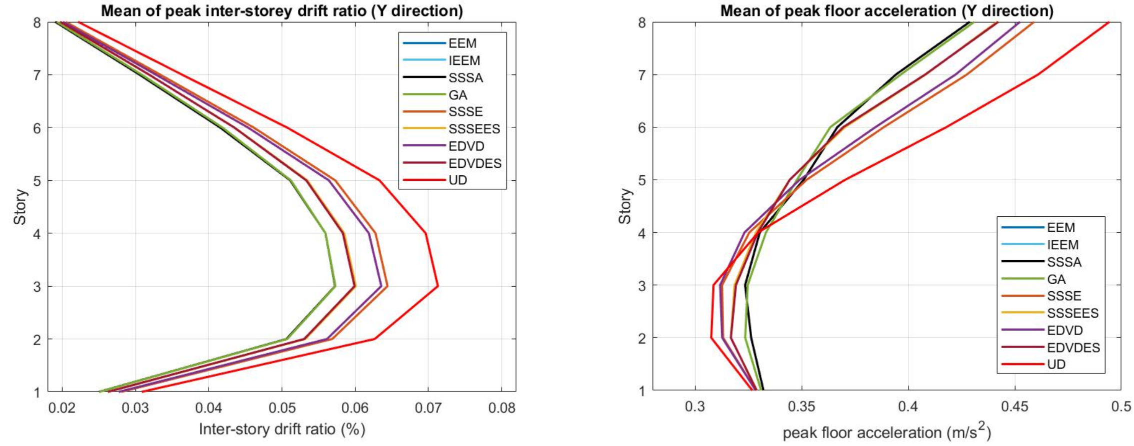

Figure 11: Mean of peak inter-storey drift ratio and floor acceleration in the y-direction of all methods

5 CONCLUSIONS

A suite of ground motions has examined the effectiveness of EEM and IEEM compared to existing methods. The study indicates that EEM and IEEM can minimise EDPs at the same level as SSSA and GA do. Among all distribution methods which do not require time history analyses and any iteration, EDVDES can obtain the minimised EDPs close to that of EEM, IEEM, SSSA, and GA. Moreover, even though IEEM, SSSA, GA can mitigate the peak IDR in the x-direction, they are not capable of controlling peak floor acceleration on the top of the building. However, it is noteworthy that IEEM, SSSA, and GA are still able to mitigate the mean of PFA compared to UD.

Table 3: Mean and standard deviation of maximum peak IDR of all methods under all ground motions in the x-direction

Peak IDR (%)	**EEM**	**IEEM**	**SSSA**	**GA**	**SSSE**	**SSSEES**	**EDVD**	**EDVDES**	**UD**
Mean	0.065	0.065	0.065	0.065	0.0716	0.067	0.070	0.067	0.078
Standard Deviation	0.058	0.057	0.057	0.057	0.066	0.061	0.065	0.060	0.073

Table 4: Mean and standard deviation of maximum PFA of all methods under all ground motions in the x-direction

Peak PFA (m/s^2)	**EEM**	**IEEM**	**SSSA**	**GA**	**SSSE**	**SSSEES**	**EDVD**	**EDVDES**	**UD**
Mean	0.450	0.479	0.479	0.479	0.467	0.450	0.462	0.449	0.494
Standard Deviation	0.357	0.367	0.367	0.367	0.397	0.370	0.392	0.368	0.434

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