NEW ZEALAND SOCIETY FOR
EARTHQUAKE ENGINEERING

Advances in floor diaphragm in-plane modelling using truss elements

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ABSTRACT

Different methods to model building floor diaphragm in-plane stiffness with truss elements are compared to identify the method most suitable for analysis in design. The methods considered include (i) Elastic truss, (ii) Diagonal compression-only truss members with compression/tension orthogonal members, (iii) Diagonal compression-only truss members, considering reinforcement as orthogonal members in tension and concrete in compression, (iv) Diagonal compression-only truss members with tension-only orthogonal members, and (v) Diagonal compression-only truss members with compression/tension orthogonal members based on Section C5 of "The Seismic Assessment of Existing Buildings" recommendations (2017). The truss elements were placed in square blocks and are compared with FEM in terms of global stiffness and diaphragm internal load path.

It was found that, truss model with diagonal compression-only and orthogonal compression/tension elements (representing un-cracked diaphragm properties), and truss model with diagonal compression-only and orthogonal reinforcement tension/concrete compression members (representing cracked diaphragm properties) satisfied the criteria of providing simple outputs usable for design and reasonable accuracy. They were recommended for use in diaphragm design.

Keywords: *Diaphragm modelling; Truss element modelling; Earthquake resistant building.*

1 INTRODUCTION

Until around 1980, building diaphragms were generally in-situ reinforced concrete slabs with high in-plane strength and stiffness. Increasingly, from around 1980 to the present day, diaphragms in NZ reinforced concrete framed buildings have often comprised a thin lightly reinforced topping slab on precast units. More recently thinner composite slabs, consisting of concrete on cold formed steel decking, all supported on a grid-work of steel beams, have increased in dominance, especially for steel buildings. Such composite slabs are economical as there is no need for other formwork, there is a high construction speed, and the light weight reduces the sizes of other frame elements and foundations. Despite the benefits, thin diaphragms may be flexible both in-plane and out-of-plane. The in-plane flexibility may change the seismic response of the structure and the distribution of force between vertical lateral force resisting (VFLR) elements. This may cause diaphragm distress or damage at connections or other locations within the load path during an earthquake event.

Since diaphragm damage may result in collapse of the diaphragm itself, life loss, and instability of the frame, it is important that it not fail. From the discussion above, it may be seen that in order to prevent undesirable diaphragm behaviour, the in-plane diaphragm behaviour must be understood.

The aim of this research is to quantify the accuracy of five truss modelling methods for diaphragms so that appropriate methods can be used with confidence in design. This is done by seeking answers to the following questions:

1. What is truss modelling method for diaphragms?
2. What are some different truss modelling methods?
3. Which truss modelling method can better represent diaphragm in-plane stiffness and internal load path?

2 TRUSS ELEMENT MODELLING (A VARIATION OF THE STRUT AND TIE METHOD)

In the Strut and Tie method, compression struts and tension ties are placed throughout the floor slab to develop a truss system of acceptable force paths. The Strut and Tie method can be used for general floor plan layouts with different irregularities such as openings in the diaphragm (Gardiner, 2011 and Bull, 2004). In addition, it can provide information that is directly useful for the designer.

The idea of replacing the continuous material of an elastic body with a framework of truss elements according to a definite pattern and suitable element properties was introduced by Hrennikoff (1941). Hrennikoff proposed the elastic framework method for solving two-dimensional stress problems. If framework elements are square, placed in a diagonal configuration (⊠), and the Poisson's ratio of the floor diaphragm material is 0.33, and diagonal elements carry force in tension and compression, then if appropriate element properties are selected and the size of the unit pattern of truss is sufficiently small, then the results of the framework method are the same as exact differential equation solution.

3 DIFFERENT TRUSS ELEMENT MODELLING TYPES

Generally, the truss method first was developed considering the elastic behaviour of a plate member, and it was assumed that all truss elements can carry tension and compression forces to satisfy the equilibrium and compatibility equations. However, for modelling a diaphragm consisting of concrete material, the tension behaviour of the concrete (cracking) should be taken into account to achieve more realistic results. Therefore, different modelling techniques are introduced and compared here to find out the best method for modelling concrete diaphragms. The modelling methods considered on a square truss mesh unit include the following:

1. Elastic truss modelling
2. Compression-only diagonal members with elastic orthogonal members (compression and tension)
3. Compression-only diagonal members with considering reinforcements as orthogonal members (concrete compression, tension reinforcement)
4. Compression-only diagonal members with tension-only orthogonal members
5. Compression-only diagonal members with elastic orthogonal members (based on recommendations from NZSEE (2017) Seismic Assessment of Existing Buildings - Section C5)

Only diamond ◈ configurations are considered here as these provide information more suitable for design than diagonal ⊠ configurations (Alizadeh et al. 2018).

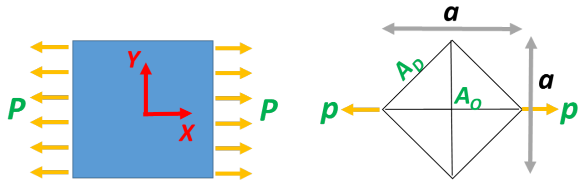
3.1 Elastic truss modelling (all members carry both tension and compression)

This method has been described comprehensively by Hrennikoff (1941). It accurately estimates the stiffness and displacements if the assumptions described above are met and the orthogonal and diagonal member cross-section areas are $A_O=0.75at$ and $A_D=0.53at$ respectively, where a is the dimension of framework pattern and t is the plate thickness. Here it is placed in a diamond ◈ configuration, rather than the original diagonal ⊠ configuration used by Hrennikoff.

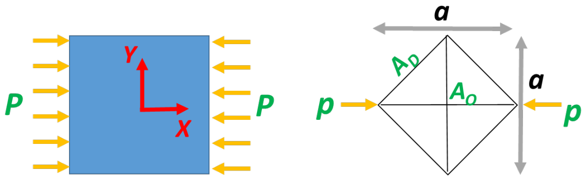
3.2 Compression-only diagonal members with compression/tension orthogonal members

The elastic truss model can effectively model the shell element behaviour. However, it does not provide good information about the actual strut compression forces and tension tie forces because the diagonal members are assumed to carry both tension and compression forces. Therefore, the truss element method with compression-only diagonal members (which represents concrete struts) may be used for finding more realistic load paths through the diaphragm.

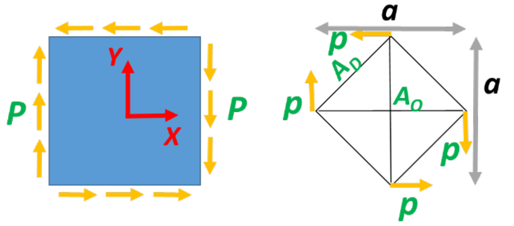
For calculating truss element cross-section area loading conditions in Figure 1 are considered: 1) uniaxial tension, 2) uniaxial compression and, 3) pure shear.



a) Normal tension force per unit length, *P* in the X-direction



b) Normal compression force per unit length, *P* in the X-direction



c) Pure shear force per unit length, *P*

Figure 1: General loading conditions for determining framework properties, p is the normal edge force to each element where p=Pa

Using equilibrium and compatibility conditions the cross-section area of the truss elements can be obtained. Note that these areas are different from those assuming all fully elastic elements as described by Hrennikoff (1941).

The truss element cross-section areas for diagonal members in this case is equal to, $A_D=at$ and for orthogonal members in tension and compression are $A_{OT}=at$ and $A_{OC}=0.585at$ respectively.

3.3 Compression-only diagonal members with reinforcement tension/concrete compression orthogonal members

In this case it is assumed that all the tension forces is carried by slab reinforcement in a fully cracked situation. Therefore, the tension cross-sectional area of the orthogonal members, A_{OT}, is equal to the steel reinforcement area, A_s, within the element width as $A_{OT}=A_s$.

Using equilibrium and compatibility conditions for loading cases shown in Fig. 1, the truss element cross-section areas for diagonal members can be obtained as, $A_D=0.725at$ and for orthogonal members in tension and compression are $A_{OT}=A_s$ and $A_{OC}=0.97at$ respectively.

3.4 Compression-only diagonal member, tension-only orthogonal member

Another potential method for truss element modelling is to consider compression-only diagonal members and tension-only orthogonal members. This method seems to be easier to use because diagonal members act in compression only and orthogonal members in tension-only. Therefore, the results may be easier to interpret for designer. However, this method does not satisfy compatibility equations and cannot be considered as a possible modelling method.

3.5 Compression-only diagonal members with compression/tension orthogonal members (based on NZSEE (2017) Seismic Assessment of Existing Buildings, Section C5 recommendations)

In these NZSEE (2017) guidelines, the truss model is recommended to obtain diaphragm design actions based on the practice note prepared by Holmes consulting group, (2015). This document was prepared based on Hrennikoff's (1941) study. The effective width of truss elements is recommended for orthogonal and diagonal members equal to $A_O=0.53at$ and $A_D=0.75at$ respectively. Where a is the dimension of framework pattern and t is the plate thickness. Diagonal members carry compression forces only (which is different from Hrennikoff (1941) where elements carry compression and tension forces). This guideline specifies a diagonal ⊠ rather than a diamond ⟐ configuration.

Also, this guideline provides some recommendations regarding the effective thickness of the truss elements. For in situ slabs and flat slabs, the combined thickness of the topping and units (if present) parallel and transverse to the units (if present) was considered. And for steel profile composite floors it is suggested that, the average of the flange and web cross-section areas for elements parallel to the webs. The thickness of the flange for truss members transverse to the webs was also suggested.

4 COMPARISON OF DIFFERENT TRUSS ELEMENT MODELLING TECHNIQUES

In order to compare the truss modelling methods, the cantilever beam-plate shown in Figure 2 was modelled with the different techniques. The results are compared in terms of global stiffness and internal forces on specific plate elements.

4.1 Stiffness investigation

Aspect ratios, L/W, of 1, 2, 4, and 10 are investigated and the stiffnesses of Model 1-5 are presented in Table 3. Figure 10 shows the geometry of the investigated model where w is equal to 1.5m and L is changing with aspect ratio. Shear force is applied at the far end of the diaphragm and the other end is fixed. The cantilever beam-plate is considered to be reinforced with DH8@250mm mesh which is equal to 0.33% reinforcement ratio. The elastic modulus of concrete, Poisson's ratio and reinforcement modulus is 32 GPa, 0.2 and 200 GPa respectively. The slab topping thickness is 50mm. The truss mesh unit dimension in all the models is equal to 250mm.

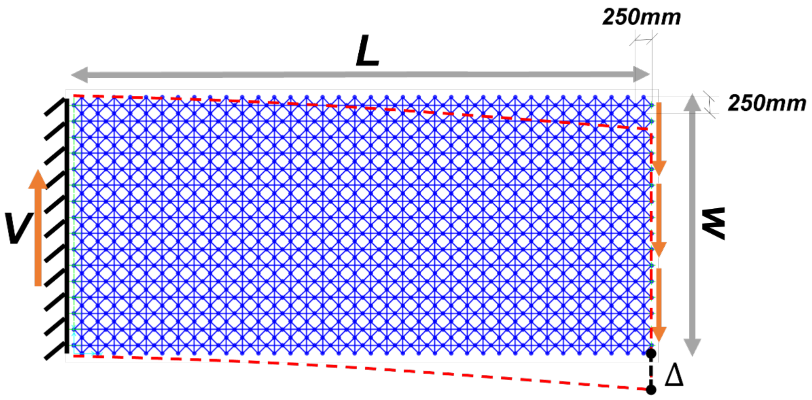


Figure. 2: Cantilever beam-plate considered for comparing truss element modelling techniques

Table 1 summarises the stiffness of all models. Also the results are plotted in Figure 3. In this figure, the stiffness of each model is normalised by the stiffness of Model 6, shell elastic, which is from a finite-element model of a flat concrete slab considering shell effects. It is considered to be the most accurate elastic (non-cracking) solution.

Model 1, truss elastic, gives the closest result to the shell model (Model 6) in terms of stiffness. The result is not exact due to the Poisson's ratio of 0.2 for concrete, rather than 0.33 for Hrennikoff), the diamond configuration, and the limited number of elements. While it is still the most accurate, it is not easy to implement in design because diagonal members act in both compression and tension making it difficult to directly obtain compression strut and tension tie demands.

Table 1: Stiffness of investigated modelling techniques for different aspect ratios

	Stiffness (kN/mm)				
Aspect ratio (L/w)	Model 1, Truss elastic	Model 2, Diagonal C-only-1 (elastic)	Model 3, Diagonal C-only-2 (cracked)	Model 5, Diagonal C-only "C5 guideline"	Model 6, Shell elastic
1	260.41	187.05	13.44	123.76	299.94
2	48.73	40.63	2.80	30.69	52.19
4	6.93	6.35	0.47	5.22	7.22
10	0.46	0.44	0.036	0.38	0.47

Model 2, diagonal compression-only elastic, presents a good stiffness specially for large aspect ratios. Also its results are easier to investigate and obtain design parameters. Model 3, diagonal compression-only cracked, may be suitable for considering diaphragm cracking. This model considers total cracked stiffness of diaphragm and same as Model 2, diagonal compression-only elastic, and its results can easily be used for design. Model 5, diagonal compression-only "seismic assessment guideline", shows the same trends as Model 2, diagonal compression-only elastic, while it underestimates the stiffness up to 60% for aspect ratio one it reaches to nearly 80% for aspect ratio 10 and larger.

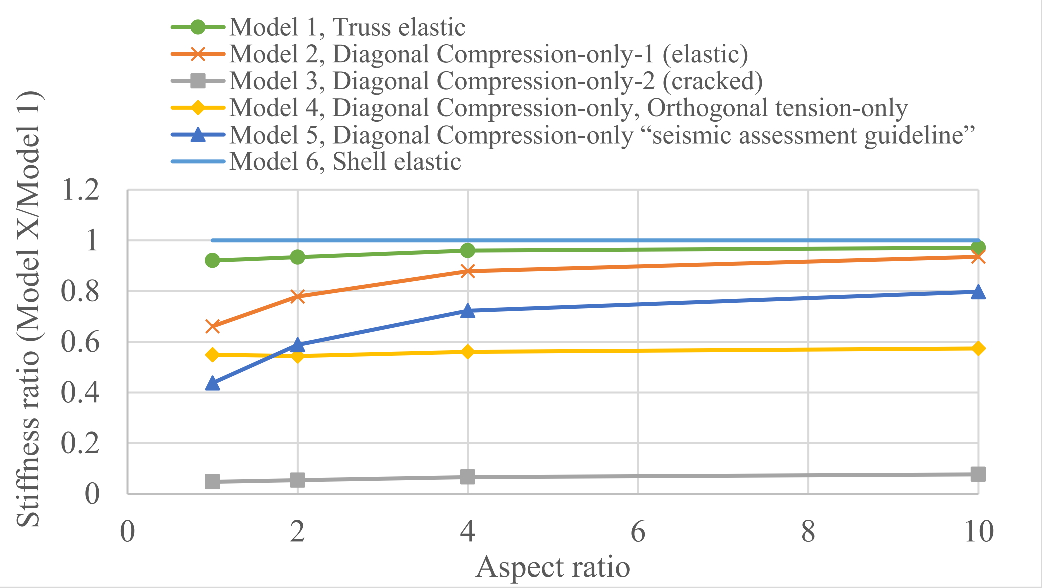


Figure 3: Stiffness ratios of different models respect to Model 6, shell elastic.

4.2 Investigating element forces in truss models

Load path methods can be used to obtain the force distribution to VLFR elements, the internal diaphragm forces, the required reinforcement, and the concrete strut axial compression demands. This can easily be achieved by using truss element modelling which directly gives the axial tension and compression demands and there is no need for considerable post-processing.

Figure 4 shows the investigated truss model with an aspect ratio of 4.0. Compression and tension forces in two horizontal orthogonal members and one diagonal member are presented in Table 2. In addition, two support reactions are also recorded to make a comparison with elastic shell model and an additional nonlinear FEM model. Here, F_{OT} and F_{OC} are orthogonal member tension and compression forces respectively. The diagonal member compression force is denoted by F_{DC}, and the support compression and tension reactions are denoted by F_{RC}, F_{Rt} respectively.

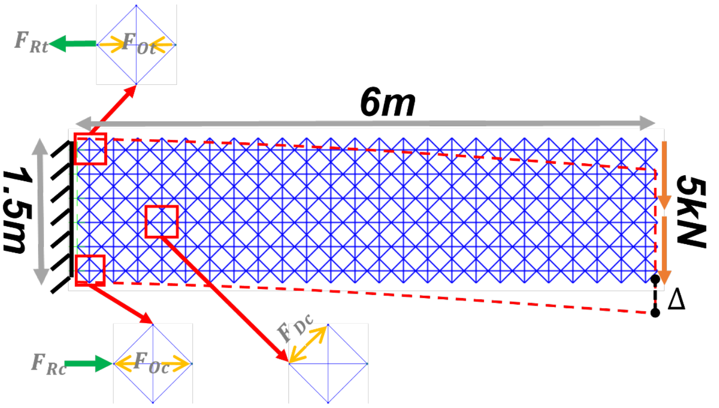


Figure 4: Truss model with aspect ratio 4 considered to investigate internal truss element forces

The nonlinear solid FEM modelling was conducted using ABAQUS. The ABAQUS model consists of detailed FEM model using 3D solid elements for concrete and one-dimensional truss elements for the reinforcement. The FIB model code for concrete structures (2010) is used to obtain the stress-strain relation for short-term loading of concrete in compression. Considering 33MPa concrete, 32 GPa elastic modulus and 0.0035 ultimate strain. A bilinear material is used for modelling reinforcement steel considering 0.01% strength hardening for better numerical convergence. Steel with $F_y=300MPa$ is used in this study.

Concrete damaged plasticity material model is used for modelling concrete in ABAQUS software. This material model assumes that the two main concrete failure mechanisms are tensile cracking and compressive crushing of the concrete material. The concrete material in ABAQUS solid elements considers $0.01f'_c$ as the minimum tensile strength of concrete as the default value. The reinforcing bars were modelled using the ABAQUS "plastic" model with isotropic hardening with von-Mises yield criterion.

The truss element forces in Model 1 are similar to the elastic shell element model (Model 6) as it was expected. The internal element forces of Model 3 show good agreement with Models 7, which means the fully cracked elastic model (Model 3) is able to estimate compression and tension demands reasonably. Models 2 and 5 show similar results with a small variation in orthogonal member compression force which is due to the difference in diagonal member cross-section areas.

Table 2: Truss element forces and support reactions

#	**Model description**	**F_{Ot} (kN)**	**F_{Oc} (kN)**	**F_{Dc} (kN)**	**F_{Rt} (kN)**	**F_{Rc} (kN)**
1	Truss elastic	12.70	12.70	1.41	17.90	17.90
2	Diagonal C-only-1(elastic)	14.40	11.50	2.15	14.47	21.80
3	Diagonal C-only-2 (cracked)	10.06	28.80	1.86	10.05	33.10
5	Diagonal C-only , NZSEE (2017) seismic assessment guideline	13.80	15.60	2.00	13.84	22.57
6	Shell elastic	--	--	1.35	18.60	18.60
7	Nonlinear FEM	--	--	0.90	9.92	32.50

5 CONCLUSIONS

This paper shows that:

1. Truss models, consisting of tension and or compression elements with different properties can be assembled in a grid to represent the in-plane behaviour of a floor diaphragm. The grid can have different shapes or aspect ratios.
2. Different models considered were elastic models, inelastic models, models considering cracking of the concrete, and those with elements placed in a diamond configuration. The stiffnesses/displacements of these were compared with elastic shell finite element analysis and with non-linear finite element analysis considering realistic diaphragm steel and concrete properties.
3. Considering both accuracy, and the usefulness of the outputs the following are recommended for use in design:
    a. Truss Model 2 with diagonal compression-only and orthogonal compression/tension elements for uncracked diaphragms,
    b. Truss Model 3 with diagonal compression-only and orthogonal reinforcement tension/concrete compression members for cracked diaphragms.

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